M_W-M_L AND M_W-M_C SEISMIC MAGNITUDE SCALING RELATIONSHIPS FOR CUBA

RELACIONES DE ESCALA DE MAGNITUD SÍSMICA M_W-M_L Y M_W-M_C PARA CUBA

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Earthquake catalogs typically report multiple magnitude types, which makes it difficult to perform comprehensive statistical analysis. Internationally, moment magnitude is the preferred reference scale for unification. This study establishes empirical relationships between M_W and two magnitudes used in Cuba: (1) amplitude-based M_L and (2) coda-duration-based M_C, as computed by the National Seismological Service of Cuba for earthquakes recorded from 1998–2022.

We evaluated linear and nonlinear regression models, including those accounting for uncertainties in the independent variable (M_L or M_C). Model parameters were estimated via standard least squares, orthogonal distance least squares, and higher-order moments regression. Using the Akaike (AIC) and Schwarz (BIC) information criteria, we identified the segmented model using orthogonal distance regression (ODR) as most recommended for both M_W–M_L and M_W–M_C relationships. These results provide a robust basis for magnitude conversions in Cuban catalog homogenization and seismicity analysis.

Los catálogos de terremotos reportan múltiples tipos de magnitud, lo que requiere una magnitud unificada para análisis estadísticos. La magnitud momento (M_W) es la escala preferida para la unificación. Establecimos relaciones empíricas entre M_W y dos magnitudes utilizadas en Cuba: M_L basada en amplitud de onda y M_C basada en duración de coda, calculadas por el Servicio Sismológico Nacional de Cuba entre 1998 y 2022.

Evaluamos modelos de regresión lineal y no lineal, incluyendo aquellos que consideran la incertidumbre en la variable independiente (M_L o M_C). Estimamos los parámetros de los modelos mediante mínimos cuadrados estándar, mínimos cuadrados ortogonales y regresión por momentos de orden superior. Mediante los criterios de información de Akaike (AIC) y Schwarz (BIC), identificamos que el modelo de regresión segmentada por distancia ortogonal es el más recomendable para las relaciones M_W-M_L y M_W-M_C. Estos resultados permiten homogenizar catálogos sísmicos cubanos y mejorar los estudios de sismicidad.

Keywords: Earthquakes in Cuba (Terremotos en Cuba), Earthquake magnitudes (Magnitudes de terremoto), Errors in physics (Errores en física), Seismicity (Sismicidad), Statistics (Estadística).

I. INTRODUCTION

The primary information contained in earthquake catalogs includes the location and magnitude of seismic events. The most common representation of earthquake size is the magnitude. Typically, catalogs report two or more types of magnitude measurements. The Cuban National Seismological Service (SSN) employs mainly the following magnitude scales: M_L (based on wave amplitude), M_C (based on coda duration) and M_W (based on seismic moment).

For statistical analyses of catalog data, such as studies of the coverage areas of seismological networks, seismicity, or hazard assessment, magnitude unification is applied to incorporate the largest possible number of observations. This process involves homogenization of all events to a single magnitude scale.

The moment magnitude M_W is considered the most representative of the tectonic effects of an earthquake and remains valid across the entire magnitude range [1]. Unlike other magnitudes, M_W does not saturate at high values, which has led to its adoption as the international reference for magnitude unification. In cases where M_W is unavailable, it can be estimated from other available magnitude types using regression-based formulas.

Magnitude conversion is a common practice worldwide [2–6]. Although several studies have examined relationships between earthquakes in Cuba [7–9], the relationships between M_W and M_L or M_C have not yet been thoroughly investigated. This study aims to establish empirical relationships between M_W and these magnitudes, providing essential tools for catalog homogenization and seismicity analysis within the country.

II. MATERIALS AND METHODS

We utilized the instrumental earthquake catalog from the SSN covering the period 1998–2022, within the region bounded by the coordinates 73.00°–84.00° W longitude and 19.00°–24.00° N latitude. The catalog was refined using the ECP package [10], which involved removing duplicates and correcting errors through cross-validation of parameters within an acceptable range. This process yielded 14,858 pairs of observations (M_W, M_L) with M_L in the range [-1.9, 6.6], and 14,328 pairs (M_W, M_C) with M_C in [-0.4, 6.2].

While the linear model has been extensively employed for converting between different earthquake magnitude types [11], a variety of other models have also been applied in this context. These include bilinear [5, 12–17], polynomial

[18–22], and exponential models [12, 23–26]. Furthermore, more complex modeling approaches have been investigated [5, 24, 27, 28]. It is important to note that while not all cited studies exclusively focus on relationships between moment magnitude (M_W) and local or coda magnitudes (M_L/ M_C), these alternative models are also applicable for such conversions.

Consequently, to explore these relationships, the following models were considered:

$$\text{Linear: } M_W = a + bM \tag{1}$$

$$\text{Segmented (bilinear): } M_W = a + bM + c\,\text{máx}(M - d, 0) \tag{2}$$

$$\text{Polynomial2: } M_W = a + bM + cM^2 \tag{3}$$

$$\text{Polynomial3: } M_W = a + bM + cM^2 + dM^3 \tag{4}$$

$$\text{Exponential1: } M_W = ae^{bM} \tag{5}$$

$$\text{Exponential2: } M_W = ae^{bM} + c \tag{6}$$

Where M can be either M_L or M_C.

Table 1 summarizes the types of regression employed, their objectives, and the conditions under which they are applied.

Table 1. Types of Regression.

Type	**Objective**	**Assumption**
Standard	Minimize the sum of squared vertical residuals	Errors are present only in the dependent variable
Orthogonal distance [29]	Minimize the sum of squared orthogonal residuals	Errors exist in both dependent and independent variables
Higher-order moments [30]	Match theoretical moments with observed data moments	Errors are present in both dependent and independent variables

In the case of linear regression via the method of moments, six different slope parameter estimators can be used [30]. When there is available information concerning error variances, estimators $\hat{\beta}_1$ to $\hat{\beta}_4$ should be used. Otherwise, if not a priori information exists, $\hat{\beta}_5$ or $\hat{\beta}_6$ shall be used. Here we applied the estimator $\hat{\beta}_5$, as follows [30]:

$$\hat{\beta}_5 = \frac{S_{xyy}}{S_{xxy}} \tag{7}$$

Where S_{xyy} and S_{xxy} are the third-order cross-moments (coskewness) for independent and dependent variables x and y. In order to use $\hat{\beta}_5$, the third sample moment must significantly differ from zero (indicating non-normality) and the sample size must be at less 50 [30]. Both conditions were satisfied: the sample size was large, and the data did not follow a normal distribution ($p < 0.001$ in D'Agostino and Shapiro-Francia tests).

To distinguish between linear models (1) fitted using the method of moments and those fitted using ODS, we called the first model Moments and the second Deming. At the same time, we retained the designation Linear in the case of the standard regression method.

When selecting the optimal model, we relied on information criteria such as AIC [31] and BIC [32]:

$$AIC = -2\ln(\mathcal{L}(\theta|data,\, model)) + 2K \tag{8}$$

$$BIC = -2\ln(\mathcal{L}(\theta|data,\, model)) + K\ln n \tag{9}$$

Where $\mathcal{L}(\theta|data, model)$ is the likelihood function of the parameters for the observed data and assumed model, K is the number of estimated parameters , and n is the sample size.

As can be seen, both AIC (8) and BIC (9) are based on the likelihood of the model, which quantifies how well the model explains the observed data; bigger likelihood means that the given model is more plausible [33]. In practice, it is convenient to work with the natural logarithm of the likelihood function, and the term $\ln(\mathcal{L}(\theta|data, model))$ in the formulas refers to its maximum value.

These criteria balance model fit with complexity. The first term in both AIC and BIC rewards how well the model explains the data, while the second term penalizes model complexity to guard against overfitting. This penalty increases with the number of estimated parameters (K) [34]. The BIC imposes a stricter penalty than the AIC, as its penalty term also incorporates the sample size n. Accordingly, for both criteria, a lower value indicates a better model.

For regressions using the least squares method, as used in this study, these measures can be calculated using the formulas presented in [34], which, keeping all terms, lead to:

$$AIC = n\left(\ln\sigma^2 + \ln 2\pi + 1\right) + 2K \tag{10}$$

$$BIC = n\left(\ln\sigma^2 + \ln 2\pi + 1\right) + K\ln n \tag{11}$$

Where K and n are the same as before, σ^2 is the estimated residual variance

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\varepsilon_i^2 \tag{12}$$

and represents the regression residuals. A crucial point is that, in addition to the model's structural coefficients, the residual variance must also be included in the count to determine the value of K. With this in mind, for models (1) to (6), K takes the values: 3, 5, 4, 5, 3, and 4, respectively.

AIC and BIC are broad metrics suitable for comparing models of various types [35]. They do not directly evaluate the model's quality but rather facilitate comparison among candidate models based on their respective values.

To facilitate this comparison, measures such as their difference AIC, BIC or their relative weights AIC_W and BIC_W [34,36] are frequently applied as follows:

$$\Delta AIC_j = AIC_j - AIC_{min} \tag{13}$$

AIC_j is AIC for model j, and AIC_{min} is the minimum of all AIC.

$$AIC_{W_j} = LAIC_j / \sum_m LAIC_m \tag{14}$$

Where $LAIC_j$ is the relative likelihood of the model j, given by

$$LAIC_j = e^{-\frac{1}{2}\Delta AIC_j} \tag{15}$$

Similarly, for BIC we have:

$$\Delta BIC_j = BIC_j - BIC_{min} \tag{16}$$

is BIC for model j, and is the minimum of all BIC.

$$BIC_{W_j} = LBIC_j / \sum_m LBIC_m \tag{17}$$

Where is calculated as,

$$LBIC_j = e^{-\frac{1}{2}\Delta BIC_j} \tag{18}$$

Since the model with the lowest AIC or BIC value is considered optimal, then it is assigned AIC or BIC of 0. Consequently, its relative likelihood (15) or (18) equals 1 and its relative weight (14) or (17) is the largest and closest to 1 within the model set.

Relative weights can be interpreted as relative preference or the degree of empirical evidence that supports the preference for one model over another [34,36].

For models accounting for uncertainty in all variables, the information criteria were computed similarly, but based on orthogonal residuals instead of vertical residuals; hereinafter denoted as AIC_O, BIC_O, AIC_{OW} and BIC_{OW}.

Although the method of moments does not consider orthogonal residuals, to compare its resulting model with others, we calculated its AIC_O, BIC_O, AIC_{OW} and BIC_{OW}.

The analyses were primarily conducted using R [37], utilizing the nls function for standard nonlinear regressions, and the onls package [38] for orthogonal distance regression (ODR). Visualization was facilitated with the ggplot2 package [39].

III. RESULTS

We compute the parameters of several regression models (linear and non-linear), relating M_W with M_L and with M_C, across two scenarios: 1) ignoring uncertainty in M_L and M_C and 2) considering uncertainty in M_L and M_C. The selection of the optimal model was guided by information-theoretic criteria (AIC, BIC) and their corresponding weights (AIC_W, BIC_W).

III.1. *M_W-M_L models that do not consider uncertainty in M_L.*

For the M_W-M_L relationships without uncertainty management (figure 1) the estimated parameters are provided in table 2.

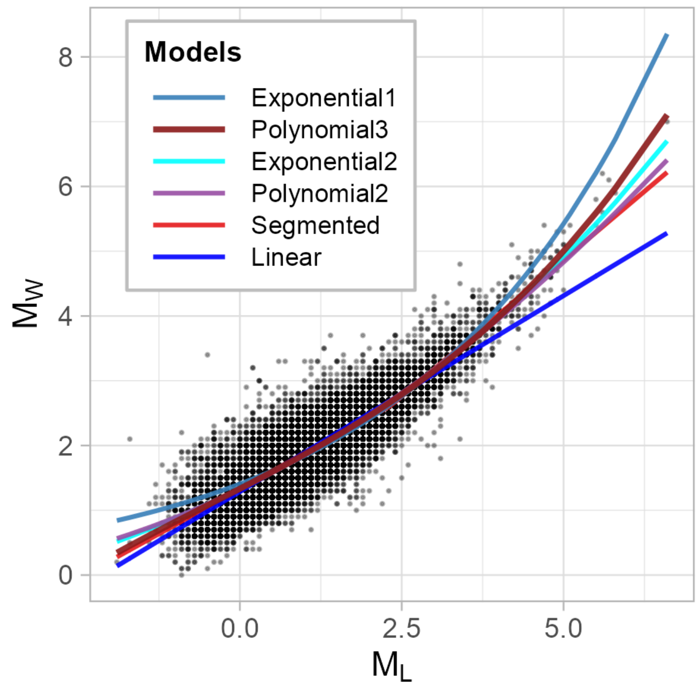

Figure 1. Graphs of M_W-M_L models that neglect uncertainty in M_L.

Table 2. Parameters of the M_W-M_L relationships (without considering uncertainty in M_L).

Model	a	b	c	d
Linear	1.287	0.605	–	–
Segmented	1.328	0.552	0.292	2.330
Polynomial2	1.329	0.485	0.043	–
Polynomial3	1.333	0.515	0.010	0.007
Exponential1	1.406	0.270	–	–
Exponential2	3.438	0.143	-2.107	–

We observed that the Polynomial3 exhibits the lowest AIC and BIC, outperforming other options due to its higher weight (AICw = 0.999, BICw = 0.991, table 3). The remaining models have minimal weights, with the Linear and Exponential1 models being the least aligned with the observations, especially at higher M_L values (figure 1, table 3).

Table 3. Information criteria for M_W-M_L models without considering uncertainty in M_L.

Model	AIC	BIC	AIC_W	BIC_W
Polynomial3	9979.6	10017.7	0.999	0.991
Exponential2	9996.7	10027.1	2.0×10^{-4}	8.9×10^{-3}
Segmented	10005.4	10035.8	2.6×10^{-6}	1.2×10^{-4}
Polynomial2	10019.4	10049.8	2.4×10^{-7}	1.1×10^{-7}
Exponential1	10513.6	10536.4	1.1×10^{-16}	2.2×10^{-13}
Linear	10531.5	10554.3	1.5×10^{-20}	2.9×10^{-17}

III.2. M_W-M_L models that incorporate uncertainty in M_L

When including uncertainty in ML (figure 2) the estimated parameters are adjusted (table 4) and the information criteria (table 5) strongly favor the segmented regression model (AIC_{OW} = BIC_{OW} = 0.871)[1], showing better behavior in the presence of errors. The model Polynomial2 becomes the second-best option, though with a considerably lower weight (AIC_{OW} = BIC_{OW} = 0.129).

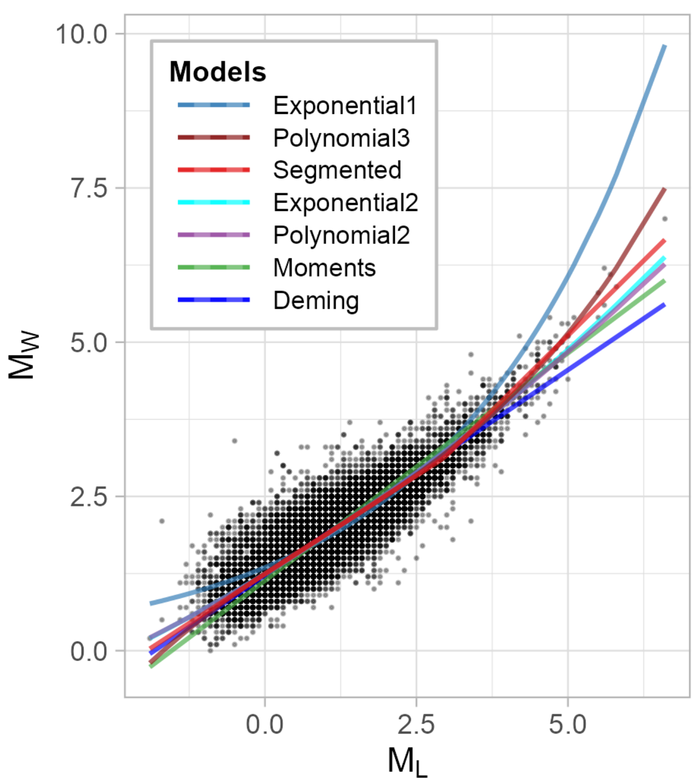

Figure 2. Graphs of M_W-M_L models that consider uncertainty in M_L.

The Exponential1 model and the linear models (Moments and Deming) demonstrated the poorest fit (figure 2, table 3). . Nonetheless, the weights assigned to Exponential2 and Polynomial2 also suggest a low likelihood of these models being optimal.

Table 4. Parameters of the M_W-M_L relationships (considering uncertainty in M_L).

Model	a	b	c	d
Linear	1.136	0.738	–	–
Segmented	1.217	0.667	–	–
Polynomial2	1.242	0.638	0.333	2.959
Polynomial3	1.245	0.593	0.025	–
Exponential1	1.348	0.301	–	–
Exponential2	7.748	0.079	-6.231	–

Table 5. Information criteria for M_W-M_L models without considering uncertainty in M_L.

Model	AIC_O	BIC_O	AIC_{OW}	BIC_{OW}
Segmented	5249.0	5287.0	0.871	0.871
Polynomial3	5252.8	5290.8	0.129	0.129
Exponential2	5324.8	5355.3	2.9×10^{-17}	1.3×10^{-15}
Polynomial2	5335.0	5365.5	1.8×10^{-19}	7.9×10^{-18}
Deming	5496.4	5519.3	3.8×10^{-54}	3.2×10^{-51}
Moments	5964.4	5987.2	3.8×10^{-156}	7.7×10^{-153}
Exponential1	6496.5	6519.3	1.1×10^{-271}	2.2×10^{-268}

III.3. M_W-M_C models that do not account for uncertainty in M_C

The M_W–M_C models that omit uncertainty in M_C (Figure 3) produced the estimates shown in table 6.

Table 6. Parameters of the M_W-M_C relationships (without considering uncertainty in M_C).

Model	a	b	c	d
Linear	1.287	0.605	–	–
Segmented	0.243	0.802	0.442	2.940
Polynomial2	0.490	0.507	0.081	–
Polynomial3	0.310	0.783	-0.046	0.018
Exponential1	0.817	0.398	–	–
Exponential2	3.045	0.186	-2.591	–

Table 7. Parameters of the M_W-M_C relationships (without considering uncertainty in M_C).

Model	AIC	BIC	AIC_W	BIC_W
Segmented	9967.7	9998.0	1.00	1.00
Polynomial3	10025.6	10063.4	2.8×10^{-13}	6.3×10^{-15}
Exponential2	10038.3	10068.5	4.8×10^{-16}	4.8×10^{-16}
Polynomial2	10052.5	10082.8	3.9×10^{-19}	3.9×10^{-19}
Linear	10052.5	10082.8	4.4×10^{-88}	1.9×10^{-86}
Exponential1	10588.3	10611.0	1.8×10^{-135}	7.8×10^{-134}

The segmented model minimizes the information criteria (table 7), achieving the highest weight (AIC_W = BIC_W = 1.00), which indicates strong evidence supporting its selection. The other models exhibit minimal relative support and are considered highly unlikely.

[1]The subscript "O "indicates that the criteria were calculated from the orthogonal residuals.

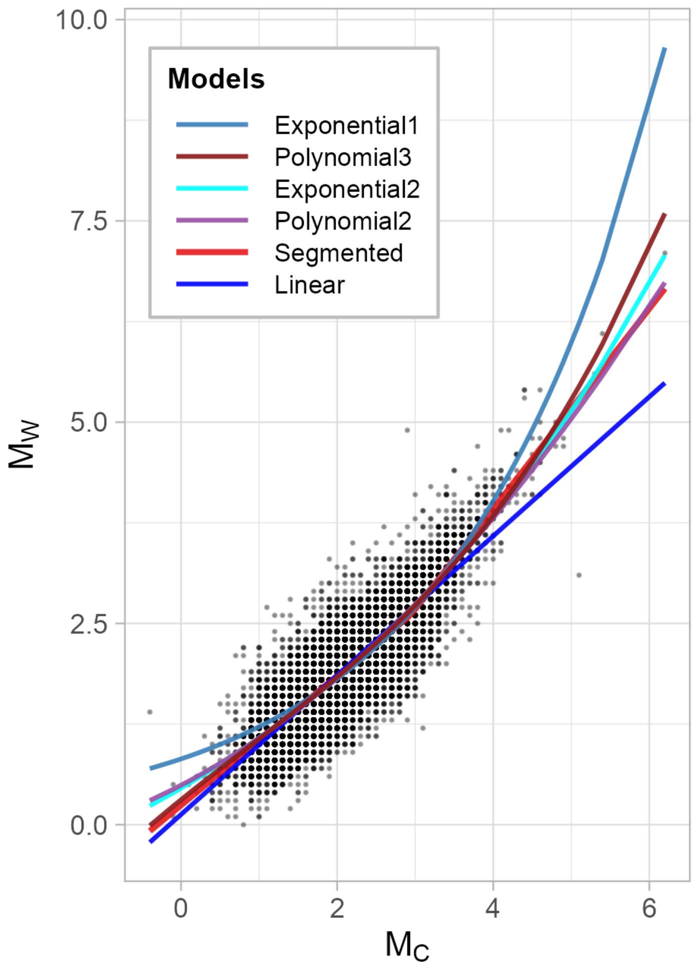

Figure 3. Graphs of M_W-M_C models that neglect uncertainty in M_C.

The segmented model minimizes the information criteria (table 7), achieving the highest weight AIC_W = BIC_W = 1.00), which indicates strong evidence supporting its selection. The other models exhibit minimal relative support and are considered highly unlikely.

III.4. M_W-M_C models that consider uncertainty in both magnitudes.

When we incorporated uncertainty into M_C (figure 4), the optimal settings changed.

Table 8. Parameters of the M_W-M_C relationships (considering uncertainty in both magnitudes).

Model	**a**	**b**	**c**	**d**
Linear	1.136	0.738	–	–
Segmented	1.217	0.667	–	–
Polynomial2	1.242	0.638	0.333	2.959
Polynomial3	1.245	0.593	0.025	–
Exponential1	1.348	0.301	–	–
Exponential2	7.748	0.079	-6.231	–

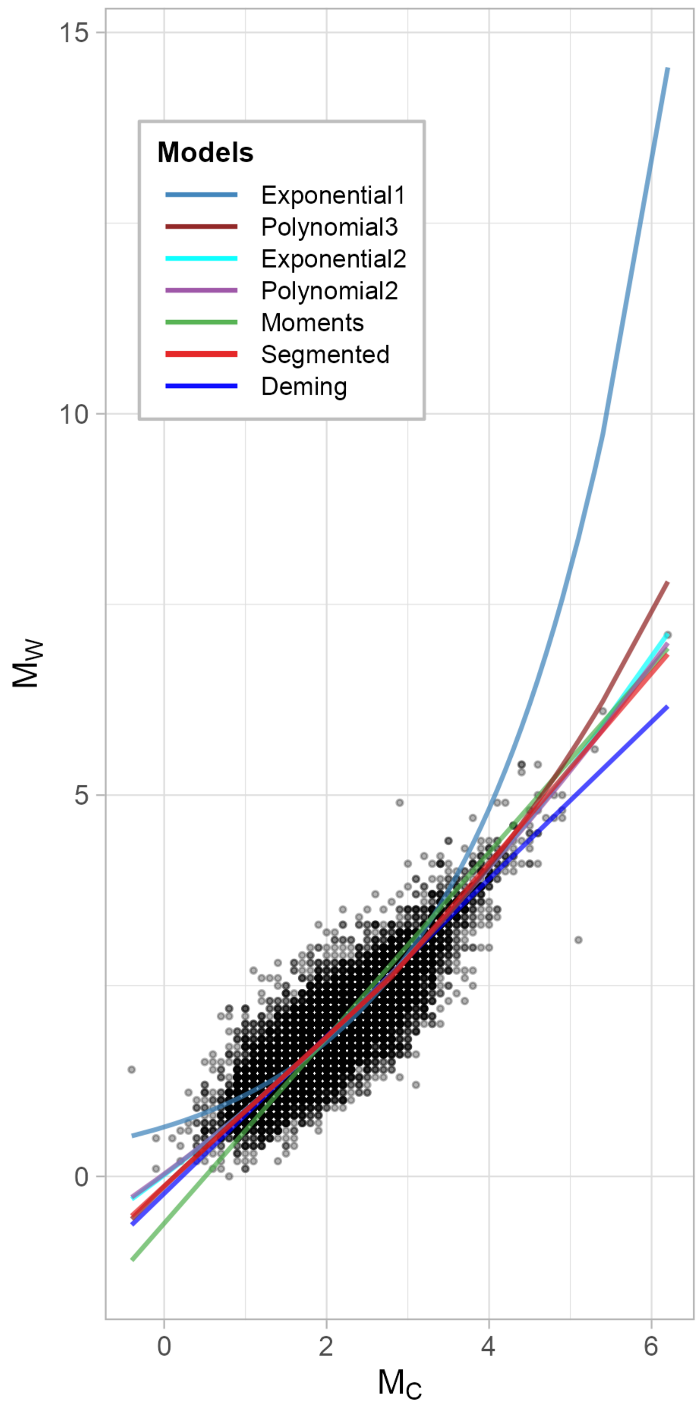

Figure 4. Graphs of M_W-M_C models that consider uncertainty in M_C.

The estimated parameters appear in table 8. The information criteria (table 9) consistently favored the segmented orthogonal model, which again outperformed all other alternatives (AIC_{OW} = 0.999 y BIC_{OW} = 0.998).

Once more, the Exponential1 model and the linear models (Moments and Deming) demonstrated the poorest fit (figure 4, table 9). Also, the weights associated with Exponential2 and Polynomial2 provide substantial evidence against these models.

Table 9. Information criteria for M_W-M_L models (without considering uncertainty in M_L).

Model	AIC_O	BIC_O	AIC_{OW}	BIC_{OW}
Segmented	5249.0	5287.0	0.871	0.871
Polynomial3	5252.8	5290.8	0.129	0.129
Exponential2	5324.8	5355.3	2.9×10^{-17}	1.3×10^{-15}
Polynomial2	5335.0	5365.5	1.8×10^{-19}	7.9×10^{-18}
Deming	5496.4	5519.3	3.8×10^{-54}	3.2×10^{-51}
Moments	5964.4	5987.2	3.8×10^{-156}	7.7×10^{-153}
Exponential1	6496.5	6519.3	1.1×10^{-271}	2.2×10^{-268}

IV. DISCUSSION

In all cases, we observed that the pure exponential model (Exponential1) and the linear models (Moments and Deming) demonstrated the poorest performance. Although the Deming regression model overall outperformed Moments, exhibited less statistical support at higher magnitudes. Conversely, the exponential model with an independent term (Exponential2) consistently yielded a better fit than the pure exponential, while the cubic polynomial (Polynomial3) outperformed the quadratic polynomial (Polynomial2).

As noted by citeCastellaro2006 the standard least squares method is inappropriate for converting seismic magnitudes because: both variables are affected by uncertainties (which contradicts the assumptions underlying least squares) and the magnitudes do not follow a Gaussian distribution.

To address these issues, citeCastellaro2006 proposed the generalized orthogonal regression (GOR). However, this approach requires previous knowledge of the ratio (η) between the standard deviations of the variables citeCastellaro2006, information that is almost always unavailable in seismic catalogs.

Moreover, GOR is not applicable to nonlinear models [17].

To incorporate the uncertainty in both variables we use:

- Higher-order moment regression [30] for linear models (which does not require knowledge of η), and
- Orthogonal distance regression (ODR) [29] for nonlinear cases. Although infrequently applied in seismology, ODR proves effective for nonlinear conversions [17]. We assume $\eta = 1$ (indicating similar errors in both variables), which is equivalent to Deming regression in linear contexts.

For both magnitude types (M_L and M_C), segmented orthogonal distance regression was found to be optimal when accounting for uncertainties. Our preferred models (with standard errors) are:

$$M_W = (1.242 \pm 0.005) + (0.638 \pm 0.004)M_L + (0.333 \pm 0.025)\,\text{máx}[M_L - (2.959 \pm 0.063), 0] \quad (19)$$

Range: $(-1.9 \leq M_L \leq 6.6)$.

$$M_W = (0.126 \pm 0.014) + (0.979 \pm 0.007)M_C + (0.266 \pm 0.022)\,\text{máx}[M_C - (2.799 \pm 0.047), 0] \quad (20)$$

Range: $(-0.4 \leq M_C \leq 6.2)$.

Our findings align with previous research when a wide range of magnitudes was used:

- For small earthquakes ($M_L < 3$), M_W exhibits a direct proportionality to 2/3 M_L [16,40].
- Both models present a change in slope (d) near M $\approx$ 3 (2.96 for M_L and 2.80 for M_C). Bilinear M_W-M_C relationships with a break point (e.g. M_C = 2.7) have also been documented [13].

IV.1. Limitations and Recommendations.

Nonlinear techniques (particularly ODR) demand more computational resources than linear approaches; however, their application is justified by their enhanced precision. A significant limitation is the absence of standard deviations in the catalog, which reduces the accuracy of methods such as ODR and GOR. We suggest incorporating these uncertainties in future catalog updates.

V. CONCLUSION

The segmented model identified through orthogonal distance regression is the most suitable for the M_W-M_L and M_W-M_C empirical relationships in Cuba, validated using information criteria (AIC and BIC).

This study provides the first nonlinear empirical relationships between magnitudes specific to Cuba, accounting for uncertainties in both dependent and independent variables.

These results provide a solid basis for magnitude conversions, catalog homogenization, and seismicity analysis within the region.

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