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# RADIOMETRY AND PHOTOMETRY: TWO VISIONS OF ONE PHENOMENON

RADIOMETRÍA Y FOTOMETRÍA: DOS VISIONES DE UN FENÓMENO

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Radiometry, the measurement of the energy of electromagnetic waves (EMW) is one of the most important fields of experimental physics, reaching all Natural Sciences. Photobiology, for instance, is based in the quantification of the action of light on biological systems. The interaction of EMW with photosensitive cells (an eye in a mammalian, for instance) is key in the understanding of the interaction of living beings with their environment. A human eye is sensitive only to a fraction of the EM spectrum, ranging approximately from 400 to 760 nm. Eye's sensitivity varies with wavelength, growing from 400 to 555 nm, where it reaches a maximum, then monotonically decreasing up to 760 nm, where it reaches a value of zero. That is the reason of the existence of two different (though related) system of units. In one hand the radiometric or energetic units, characterizing the energy of a light beam and, on the other, the photometric units, characterizing the action of EMW upon a human eye. Both systems are often confused, and the extent and exact definition of their magnitudes misinterpreted, without a precise relation among related units of both systems. This is so even in some textbooks. In the present contribution we will try to clarify all these topics. This paper could be useful in courses of Optics, Ophthalmic Optics, Metrology and design of illuminating systems.

La medición de la energía de las ondas electromagnéticas (OEM). la radiometría, es uno de los campos de la física experimental con más aplicaciones prácticas. Cuantificar la acción de las radiaciones sobre los sistemas biológicos, es central para la fotobiología. La acción de las OEM sobre las células fotosensibles (los ojos de los mamíferos, por ejemplo) es de singular importancia para la comprensión de la interacción de los organismos con el ambiente que les rodea. Los ojos son sensibles solo a un rango limitado de frecuencias de las OEM que sobre ellos influyen. El ojo humano es sensible a radiaciones entre 400 y 760 nm aproximadamente, rango que se conoce como luz visible. En ese rango la sensibilidad cambia con la longitud de onda, creciendo con esta a partir de 400 nm, alcanzando un máximo a los 555 nm para decrecer monótonamente hasta hacerse cero a 760 nm. Además de las unidades que caracterizan energéticamente la luz (llamadas radiométricas), se ha introducido otro sistema de unidades para caracterizar la respuesta del ojo humano a la luz (las magnitudes fotométricas). Ambos sistemas son a menudo confundidos, incluso en los libros de texto, mezclándose sus unidades. De igual forma sus nombres y definiciones se dan incorrectamente, sin precisarse la relación entre ambos sistemas. En la presente contribución trataremos de esclarecer estos aspectos. El artículo puede ser útil en la enseñanza de la Óptica, la Óptica Oftalmológica, la Metrología y el diseño de la iluminación.

PACS: Photodetectors (fotodetectores), 42.79.Pw, 85.60.Gz; Eye (ojo), 42.66.-p; Radiometers (radiómetros), 07.60.Dq; Color (color), 42.66.Ne; Ophthalmic Optics (óptica oftálmica), 42.66.Ct

### I. INTRODUCTION

The study of the interaction of electromagnetic waves (EMW) with matter is a major branch in physics. The incidence of electromagnetic energy on different systems provokes a plethora of phenomena having both theoretical an experimental importance. In order to quantify the action of EMW upon a system, a group of physical magnitudes has been developed, which has a common ground: measuring the power of the electromagnetic radiation that interacts with the system. But measuring only the power is not always enough: the effects depend also on the angle of incidence of the radiation upon a surface, the solid angle that form the rays exciting from a source and other elements. Some times it also depends on the wavelength of the radiation. It implies that is necessary to introduce a set of physical magnitudes that are used to characterize the light - matter interaction in different physical situations. All these magnitudes are grouped under the global term of radiometric magnitudes". Radiometry is, according to this, the measurement of the energy content of electromagnetic fields propagating through a region, and the determination of how this energy is transferred from a source, through a medium, and to a detector [1]. Normally radiometry is restricted to the range of infrared, visible and ultraviolet radiation, but it could also reach the microwave spectrum. The reader must be aware that in some areas of Astronomy they call photometry to what is called radiometry in Optics.

Radiometry has several branches, depending on the wavelength range of the EMW. Infrared radiometry is mainly (but not only) used in astronomy, due to the lower absorption of infrared radiation by the atmosphere, when compared to visible and ultraviolet radiation [2]. Another use is in the measurement of the temperature of objects and gases. Microwave radiometry is used in the measurement of EMW with frequencies (wavelengths) from 300 MHz (1 m) to 300 GHz (1 mm) [3]. It has applications in many different areas, from temperature measurement to radio broadcasting [4]. Ultraviolet radiometry measures EMW with wavelengths

from 10 nm to 400 nm [5] and has among it main purposes the determination of the doses absorbed due to sun light and clinical procedures [6].

Visible and near-visible radiometry measures the power (and related quantities) of light in a range of wavelength for which a human eye could develop visual sensations. Though the spectral range of visibility changes from person to person, it approximately reaches wavelengths from 400 nm to 760 nm, while the near visible range spans approximately from 360 - 830 nm [1, 7]. The objective measurement of these EMW is the same in all range of wavelengths and consists in determining the output of a detector, calibrated previously at different energies and wavelengths. But the human eye has different physiological responses to different wavelengths. Its sensitivity reaches a maximum at 555 nm and decreases to zero for wavelengths bellow 400 nm and above 760 nm, approximately. Though this effect is subjective, because is related with human perception, it has a great importance for color industry, Ophthalmology, illumination design, art and other areas where the visual sensation is central. Due to this reason, a separated set of physical magnitudes has been developed to deal with the measurement of light, considering its action upon a human eye. This set is known as "photometric system of units".

As important as this subject is, in physics textbooks it is most of times poorly treated, varying the name and definition of the units from one book to another. It is also omitted the way to calculate or measure the photometric quantities starting from the radiometric ones. The aim of the present contribution is to define precisely both systems of units, emphasizing on their relation and practical applications. To facilitate its use in different countries, the names of different units when defined will be given in English and in Spanish (in brackets). This paper could be useful in courses of Optics, Ophthalmic Optics, Metrology and design of illuminating systems.

### II. RADIOMETRIC MAGNITUDES

There exist two kind of radiometric magnitudes. The first one does not take into account the spectral composition of the light, measuring only its overall power or related units. By this reason these are called integral magnitudes. The second set expresses the power and related units for a narrow interval of wavelengths and are termed spectral magnitudes. We will use in what follows the recommendations of the International Standard Office of using the suffix e (for energetic) to distinguish the radiometric units. Also the spectral units are distinguished by a suffix  $\lambda$ , though sometimes the dependence with the wavelength could be specified putting  $\lambda$  into brackets. Another specification that is rarely used is the suffix  $\Omega$  for directional units. Readers must be aware of the fact that all the formulae developed below are valid in the approximation of geometrical optics.

Integral Radiometric Magnitudes: The fundamental radiometric magnitude is the radiant energy (energía radiante) and is defined as the energy of the EMW. In the International System of Units is measured in Joules (J). The common symbol is  $Q_e$ , though other symbols (as  $W_e$  or  $E_e$ )

are also used in the literature. Derived from this magnitude are defined:

- Radiant Flux  $\Phi_e$  (Flujo de energía radiante o flujo radiante): Radiant energy emitted, reflected, transmitted or received per unit time, and is measured in watt (W). This is the power transported by the EMW. If we are dealing with an isotropic source (emitting equal amounts of energy per second in all directions) it is possible to determine the radiant flux measuring the energy arriving to an unit area perpendicular to the flux, and dividing this amount by the time the process takes. If, on the contrary, the flux is anisotropic it is necessary to divide the illuminated field in areas small enough to consider the radiant flux  $d\Phi_e$  as uniform, being the total flux the sum of its value over all the differential areas:  $\Phi_e = \int_S d\Phi_e$ .
- Radiant intensity  $E_{e,\Omega}$  (Intensidad radiante): Is the radiant flux emitted, reflected, transmitted or received, per unit solid angle. As the suffix indicates, it is a directional quantity. Is measured in watt per steradian (W·sr<sup>-1</sup>). This unit is of paramount importance for defining the photometric units. Remember, the suffix  $\Omega$  could be omitted. The luminous flux of a source is a magnitude that characterizes it, though it could be increased or decreased by optical means: focusing the light of the source with convergent lenses will increase  $E_e$ . An HeNe laser, for instance, has a small radiant flux (is a low power light source), but has a huge radiant intensity, due to its extremely focused beam.
- *Irradiance*  $I_e$  (Irradiancia) Radiant flux received by a surface per unit area. Its unit is watt per square meter  $(W \cdot m^{-2})$ . In Optics, this is the fundamental observable, and is calculated as the average value of the modulus of the Poynting vector in a time lapse much larger than the period of the EMW:  $I_e = \langle |\vec{S}| \rangle_{\tau}$ ,  $\tau << T$ . In some textbooks is erroneously termed intensity. For a point like source, the irradiance decreases with the square of the distance between the source and the point where it is measured.
- Radiance  $L_{e,\Omega}$  (Radiancia) Radiant flux emitted, reflected, transmitted by a surface, per unit solid angle per unit projected area. Measured in watt per steradian per square meter (W·sr<sup>-1</sup>·m<sup>-2</sup>). If the source is not isotropic, the radiation field most be divided in differential solid angles, small enough to consider the radiance constant, and then integrate over all the illuminated region.
- Radiosity  $J_e$  (Radiancia emitida) Radiant flux leaving a surface per unit area. It includes the emitted, reflected and transmitted electromagnetic radiation. Its unit is watt per square meter ( $W \cdot m^{-2}$ ).
- Radiant exitance  $M_e$  (Excitancia radiante o luminosidad) Radiant flux emitted by a surface, being the emitted component of the radiosity. Is obviously measured in  $W \cdot m^{-2}$ .

These are the most frequently used magnitudes. For a complete list of radiometric units, the reader is invited to check reference [8,9].

Spectral radiometric magnitudes: This set of magnitudes is related with the distribution of the energy transported by the EMW along the wavelength (or frequency) spectrum spanned by it, the so called spectral distribution. In order to characterize this distribution, a density function has to be defined. There are two possibilities, the first one is to determine the density for an infinitesimal interval of wavelengths ( $\mathrm{d}\lambda$ ), the second for an infinitesimal interval of frequencies ( $\mathrm{d}\nu$ ). As  $\lambda \cdot \nu = c$  both magnitudes are related by the expression

$$d\lambda = \frac{c}{v^2} d\nu, \tag{1}$$

where a minus sign has been ignored; it only indicates that  $\lambda$  increases when  $\nu$  diminishes.

The first spectral magnitude is the *spectral flux* (densidad espectral de flujo) defined as the radiant flux per unit frequency or wavelength. The first is defined as

$$\Phi_{e,\nu} = \Phi_e(\nu) = \frac{\mathrm{d}\Phi_e}{\mathrm{d}\nu},\tag{2}$$

measured in W⋅Hz<sup>-1</sup>.

The radiant flux per unit wavelength is defined as:

$$\Phi_{e,\lambda} = \Phi_e(\lambda) = \frac{d\Phi_e}{d\lambda},\tag{3}$$

measured in  $W \cdot m^{-1}$ . In some books it is often used the (incorrect) form W/nm. Note that this last form, though expresses correctly the definition, is not correct from the point of view of the standard organizations, as expressed in Ref. [10].

Other spectral magnitudes are:

- Spectral intensity  $E_{e,\Omega,\nu}$  or  $E_{e,\Omega,\lambda}$  (Intensidad radiante espectral) is the radiant intensity per unit frequency or wavelength. The first one is measured in W·sr<sup>-1</sup>·Hz<sup>-1</sup>. The latter is commonly measured in W·sr<sup>-1</sup>·m<sup>-1</sup> or (incorrectly) W·sr<sup>-1</sup>·nm<sup>-1</sup>.
- Spectral irradiance  $I_{e,\nu}$  or  $I_{e,\lambda}$  (Irradiancia espectral) Irradiance of a surface per unit frequency or wavelength. The units are  $W \cdot m^{-2} \cdot Hz^{-1}$  and  $W \cdot m^{-3}$ , which means watt per unit area per unit wavelength, not watt per unit volume.
- Spectral radiance  $L_{e,\Omega,\mathcal{N}}$  or  $L_{e,\Omega,\lambda}$  (Radiancia espectral) Radiance of a surface per unit frequency or wavelength. The latter is commonly measured in W·sr<sup>-1</sup>·m<sup>-2</sup>·nm<sup>-1</sup> or in the more appropriate form W·sr<sup>-1</sup>·m<sup>-3</sup>. In terms of frequency the units are

 $W \cdot sr^{-1} \cdot m^{-2} \cdot Hz^{-1}$ .

■ Spectral radiosity  $J_{e,v}$  or  $J_{e,\lambda}$  (Radiancia emitida espectral) Is the radiosity of a surface per unit frequency or wavelength. Units are W·m<sup>-2</sup>·Hz<sup>-1</sup> or W·m<sup>-3</sup>.

The reader could derive other spectral quantities following the definitions above. It is important to recall that the suffix should be used only to avoid confusions. If the context is clear, suffix could be eliminated. It is usually preferred to write the wavelength or the frequency dependence of the magnitude as a function instead of a suffix ( $I_e(\lambda)$  instead of  $I_{e,\lambda}$ , for instance).

II.1. Relation between integral and spectral radiometric magnitudes

As far as the spectral quantities are the value of the quantity in a narrow (infinitesimal) range of frequencies or wavelengths, they act as a density function, so the relation (using as an example the irradiance) is

$$I_e(\lambda) = \frac{\mathrm{d}I_e}{\mathrm{d}\lambda}.\tag{4}$$

Due to the above mentioned relation  $\lambda \cdot \nu = c$ , it is easy to obtain that

$$I_e(\lambda) = \frac{v^2}{c} I_e(\nu),\tag{5}$$

which is equivalent to

$$\lambda I_e(\lambda) = \nu I_e(\nu). \tag{6}$$

Starting from the above equations, the following useful relations are easily obtained

$$I_e = \int_0^\infty I_e(\lambda) d\lambda = \int_0^\infty I_e(\nu) d\nu, \tag{7}$$

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$$I_e = \int_0^\infty \lambda I_e(\lambda) d \ln \lambda = \int_0^\infty \nu I_e(\nu) d \ln \nu.$$
 (8)

Similar procedure could be applied to all the radiometric units. Eqs. (4-8) were derived for vacuum. In another medium with refraction index n, (1) changes to

$$d\nu = \frac{c}{n\lambda^2} d\lambda,\tag{9}$$

This correction must be introduced to other relations.

### III. HUMAN VISION

The process of human vision is very complex, and includes physical, physiological and psychological phenomena. The physical part is the path of light rays through the cornea, a limiting aperture (iris), the lens or crystalline and the vitreous humor, to be ideally focused on the retina. There, the light energy is transformed by a complex system of biochemical reactions, occurring in the sensitive cells, into a nervous signal that travels to the brain by the optical nerve.

In the visual cortex of the brain, the signals are interpreted according to the previous experience of the subject, to understand the forms of the objects as well as the contexts where they are.

A human eye does not perceive equally all the wavelengths contained in a light beam arriving to retina. A normal eye has its maximum sensibility for monochromatic light with wavelength  $\lambda_m = 555$  nm ( $\nu_m = 5.401 \cdot 10^{14}$  Hz). It means that two monochromatic beams having equal radiant flux, one of them with  $\lambda_1 = \lambda_m$  and the other with  $\lambda_2 \neq \lambda_m$  would be perceived differently by a normal eye, in such a way that the beam with  $\lambda_m$  would be seen as more bright. The larger the difference between  $\lambda_m$  and  $\lambda_2$ , the fainter is seen the later beam, until for a limiting value of wavelength the eye looses its sensitivity: the light does not provoke a visual sensation. These limits are different from person to person and change with age, but conventionally are located at 400 nm (below which is the ultraviolet zone) and 760 nm (above which is infrared). The so called enhanced visual zone is located between 360 and 830 nm [1].

An experiment to quantify this effect could be schematically described as follows: the person subject of the experiment sees with only one eye the beams  $\lambda_1$  and  $\lambda_m$  alternatively. Let us suppose that initially the radiant flux of both beams is equal,  $\Phi_{e,i}(\lambda_1) = \Phi_e(\lambda_m)$ . It is obvious that the first beam is perceived fainter by the eye. Then, its radiant flux is increased until (for a given value  $\Phi_{e,f}(\lambda_1)$ )the subject finally declares that sees both beams equally intense. The value of  $\Phi_{e,f}(\lambda_1)$  is registered, and the experiment is repeated for another wavelength. At the end of the experiment there is a table of values of radiant flux for different  $\lambda$  at which these radiations are seen equally intense when compared with  $\lambda_m$ . The eye sensitivity function (función visibilidad), also known as luminous efficiency function is defined as

$$V(\lambda) = \frac{\Phi_{e}(\lambda_{m})}{\Phi_{e,f}(\lambda)}.$$
(10)

It is obvious from the definition that  $V(\lambda) \leq 1$ , being equal for  $\lambda = \lambda_m$ . The actual procedure to construct  $V(\lambda)$  includes the repetition of this experiment for a great number of young healthy eyes. The average values has been declared by agreement as the photopic spectral luminous efficiency function by the International Committee of Illumination (or CIE, the French acronym of Commission Internationale de l'Eclairage). These values are periodically rectified to include new experimental data.

The word photopic indicates vision in bright light, as opposed to scotopic, related with vision in dim light. The cells of the retina responsible for this photopic response are called cones. There are three different types of cones, having maximum sensitivity in the range of red, green and blue, respectively. From the combined response of the three types of cells, the brain determines the color of incident light.

The retina also contains another type of photosensitive cells, the rods. Under low light, cones are almost blind and only rods are responsible of vision. The curve constructed in this illumination condition is known as scotopic efficiency function. The maximum of this curve is shifted to lower wavelengths (approximately 507 nm). In an intermediate range of irradiances (the so called mesopic vision) does not exists an accepted standard.

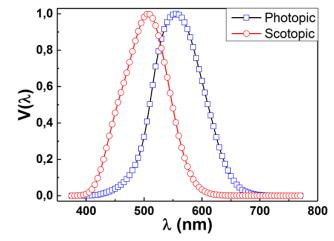


Figure 1. Photopic and scotopic eye sensitivity curves for a young healthy eye (CIE standard observer, see text).

In Fig. 1 the photopic and scotopic average response curves of a normal eye (the so called CIE standard observer) are represented. It is important to remember that it is not an analytic elementary function; the continuous line is only a spline connecting the isolated measured points. In the appendix the reader could find a Table of both photopic and scotopic eye efficiency functions.

## IV. PHOTOMETRIC SYSTEM OF UNITS AND THEIR MEASUREMENT

When the measurements of light beams is focused in their visual effect, it is obvious that their energetic description is not enough. New magnitudes have to be introduced in order to correctly describe the interaction light - eye. The set of units are analogous to energetic ones, only are weighted by the wavelength dependence of eye response  $V(\lambda)$ .

### IV.1. Photometric magnitudes

The fundamental magnitude in photometry is the *luminous intensity*  $I_v(\lambda)$  (intensidad luminosa). It is defined as the radiant intensity emitted by a light source, weighted according to the visual sensation it provokes. Its unit is named candela (also candela in Spanish, it is a fundamental unit of the International System), being one candela (not candle, the name of the unit has changed) the luminous intensity in a given direction of a source emitting monochromatic radiation of 540 THz (555.016 nm) having a radiant intensity of 1/683.002 W/sr.

Note that the definition does not use the maximum of  $V(\lambda)$ , but a very close wavelength. The suffix v is used for all photometric units to avoid confusion. The candela is abbreviated as cd. Usually the constant in the definition is rounded to 683.

The second photometric magnitude is the *luminous flux*  $\Phi_v(\lambda)$  (flujo luminoso). This is the radiant flux weighted according to its visual sensation. It is the product of the luminous intensity by the solid angle it is emitted. Its unit is named lumen (lm), which relates to candela as

$$\Phi_v(\lambda) = I_v(\lambda) \cdot \Omega, \qquad \to \qquad 1 \text{lm} = 1 \text{cd} \cdot \text{sr}.$$
 (11)

Comparing the definitions of luminous intensity and luminous flux, it is easy to see that a light beam with  $\nu=540$  THz having a luminous flux of 1 lm has a radiant flux  $\Phi_e=1/683$  W. It means that one Watt of this light creates a luminous flux of 683 lm. As far as the luminous flux is easier to measure than the luminous intensity, (11) is used to define the standard of the unit candela, as will be seen below.

What if the wavelength is another, say  $\lambda \neq \lambda_m$ ? In this case the visual sensation will be lower, by a factor determined by the sensitivity function. From Eq. (10)

$$\Phi_v(\lambda) = V(\lambda) \cdot \Phi_e(\lambda_m) = 683 \cdot V(\lambda) \cdot \Phi_e(\lambda). \tag{12}$$

Note that the constant 683 in (12) has dimensions of  $\text{Im} \cdot \text{W}^{-1}$ . It is called the maximum spectral luminous efficacy (eficacia luminosa espectral máxima) of radiation for photopic vision, Km;ph = 683  $\text{Im} \cdot \text{W}^{-1}$ . In situations of low illumination, the scotopic constant  $K_{m,sc} = 1700 \, \text{Im} \cdot \text{W}^{-1}$  has to be used. For the intermediate or mesopic regime no standard value has been defined.

Other photometric magnitudes are:

• *Illuminance*  $E_v$  (Iluminación) Luminous flux per unit area arriving at a surface perpendicular to it.

$$E_v = \frac{\Phi_v}{S}.\tag{13}$$

The unit is named lux (lx).  $1 \text{ lx} = 1 \text{ lm} \cdot \text{m}^{-2}$ . This is the photometric equivalent of irradiance.

 Luminance L<sub>v</sub> (Luminancia) Is the luminous intensity per unit area emitted by a source perpendicular to its surface.

$$L_v = \frac{I_v}{S}. ag{14}$$

Its unit is some times named nitio (nt), 1 nt =  $1 \text{ cd} \cdot \text{m}^{-2}$ , though the name has fallen in disuse. This magnitude is related with the photometric magnitude radiance.

• Luminous exitance  $M_v$  (Emitancia luminosa) Luminous flux emitted per unit area of a surface perpendicular to it

$$M_v = \frac{\Phi_v}{S}. ag{15}$$

the unit is also the lux (lx). Photometric equivalent of radiant exitance.

• Luminous exposure  $H_v$  (exposición luminosa) Is the time integral of illumination, measured in  $lx \cdot s$ .

The general relation between radiometric (Rad) and photometric (Phot) magnitudes, for monochromatic light of wavelength  $\lambda$  is

(11) Phot = 
$$683 \cdot V(\lambda) \cdot \text{Rad}$$
. (16)

In case we are dealing with polychromatic radiation it is necessary to integrate the contribution of each interval of wavelengths of the spectral distribution spanned by the radiometric quantity, to obtain the photometric one.

For instance, a luminous beam with an spectral irradiance  $I_e(\lambda)$  illuminates a surface with an illuminance

$$E_v = K_m \int_{\lambda_{min}}^{\lambda_{max}} I_e(\lambda) V(\lambda) d\lambda.$$
 (17)

In (17) the integration interval extends to all the wavelengths included in the visible range  $\lambda_{min} = 400$  nm,  $\lambda_{max} = 760$  nm. Of course, the actual range could be a subset of the visible, and the limits could be narrower. There exists situations where the extended visible range should be used.

To calculate the integral of (17) it is important to remember that  $V(\lambda)$  does not have an analytic expression, so the integral must be calculated numerically, using the equation

$$E_v = K_m \sum_{i=1}^n I_e(\lambda_i) V(\lambda_i) \Delta \lambda. \tag{18}$$

If Table IV.1 is used to calculate the corresponding photometric magnitude via Eq. (18), i goes from 1 to 80, and the  $V(\lambda_i)$  are taken from the photopic or scotopic column, depending on the conditions and  $\Delta\lambda = 5$  nm. The tabulated values of  $V(\lambda_i)$  has been determined by CIE at 1 nm increments. An interpolation process must be used if a finer mesh is needed.

The relation between irradiance and illumination is particularly important, because some times you have an instrument to measure illuminance and need the value of irradiance or vice versa as in Ref. [12], where a cellphone is used to test Mallus law and to record the diffraction pattern of a green laser monochromatic light through a thin long slit. In the second case it is easy to test, applying (16) that  $I_e(\lambda_{laser}) \propto E_e(\lambda_{laser})$ . As long as the equation that describes the irradiance distribution could be written in terms of the irradiance of the light diffracted according to a given angle  $\varphi$  divided by the irradiance of the central maximum, is simple to find that  $I_e(\varphi)/I_0 \propto E_e(\varphi)/E_0$ .

In case of a polychromatic source, there are two possibilities, firstly, the light emitted by the source is a superposition of some discrete spectral lines, a lamp of sodium or mercury, for instance. For each of the discrete wavelengths the irradiance is proportional to the illuminance, being the integral quantities also proportional.

The second possibility occurs when the light source emits a continuous spectrum, and the light does not travel through a dispersing system, as in Ref. [12] when studying Malus

law. Even in this case, considering that  $I_e(\lambda)$  is a continuous function we could apply the mean value theorem to (17) and obtain

$$I_e(\lambda_{av}) = \frac{E_v}{K_m \int_{\lambda_{min}}^{\lambda_{max}} V(\lambda) d\lambda},$$
(19)

where  $\lambda_{av} \in (\lambda_{min}, \lambda_{max})$ . This means that the action of the polychromatic source creates an illumination equal to that of a monochromatic source of wavelength  $\lambda_{av}$  and irradiance given by (19). Again the irradiance is proportional to illumination.

Table 1. Photopic and Scotopic Spectral Luminous Efficiency Functions.

$\lambda$ (nm)	Phot	Scot	λ (nm)	Phot	Scot
375	0.00002		575	0.91540	0.16020
380	0.00004	0.00059	580	0.87000	0.11212
385	0.00006	0.00111	585	0.81630	0.08990
390	0.00012	0.00221	590	0.75700	0.06550
395	0.00022	0.00453	595	0.69490	0.04690
400	0.00040	0.00929	600	0.63100	0.03315
405	0.00064	0.01852	605	0.56680	0.02312
410	0.00121	0.03484	610	0.50300	0.01593
415	0.00218	0.06040	615	0.44120	0.01088
420	0.00400	0.09660	620	0.38100	0.00737
425	0.00730	0.14360	625	0.32100	0.00497
430	0.01160	0.19980	630	0.26500	0.00334
435	0.01684	0.26250	635	0.21700	0.00224
440	0.02300	0.32810	640	0.17500	0.00150
445	0.02980	0.39310	645	0.13820	0.00101
450	0.03800	0.45500	650	0.10700	0.00068
455	0.04800	0.51300	655	0.08160	0.00046
460	0.06000	0.56700	660	0.06100	0.00031
465	0.07390	0.62000	665	0.04458	0.00021
470	0.09098	0.67600	670	0.03200	0.00015
475	0.11260	0.73400	675	0.02320	0.00010
480	0.13902	0.79300	680	0.01700	0.00007
485	0.16930	0.85100	685	0.01192	0.00005
490	0.20802	0.90400	690	0.00821	0.00004
495	0.25860	0.94900	695	0.00572	0.00003
500	0.32300	0.98200	700	0.00410	0.00002
505	0.40730	0.99800	705	0.00293	0.00001
510	0.50300	0.99700	710	0.00209	0.00001
515	0.60820	0.97500	715	0.00148	0.00001
520	0.71000	0.93500	720	0.00105	0.00000
525	0.79320	0.88000	725	0.00074	0.00000
530	0.86200	0.81100	730	0.00052	0.00000
535	0.91485	0.73300	735	0.00036	0.00000
540	0.95400	0.65000	740	0.00025	0.00000
545	0.98030	0.56400	745	0.00017	0.00000
550	0.99495	0.48100	750	0.00012	0.00000
555	1.00000	0.40200	755	0.00008	0.00000
560	0.99500	0.32880	760	0.00006	0.00000
565	0.97860	0.26390	765	0.00004	0.00000
570	0.95200	0.20760	770	0.00003	0.00000

### *IV.2. Realization of the photometric units*

In order to experimentally define the photometric units, it is important to construct a standard. This is of paramount importance for the candela, which is a fundamental unit of the International System. There are two paths, one is

to construct a source which emits radiation with a fixed luminous intensity (a source standard) and the other is to construct a calibrated measurement system (a detectorbased standard).

The first method construct a luminous source in the following way. A cylinder filled with platinum (Pt) is heated using high frequency electric currents, until it reaches the fusion temperature of Pt, 2046.6 K. This temperature is kept constant along the cylinder. In this conditions, and with the geometry of the installation, each square centimeter of the cylinder emits 60 cd in the perpendicular direction. For a detailed description the reader is invited to see Ref. [11], epigraph 8.

The second method consist in the construction of a calibrated sensor. The sensor is illuminated by a point-like source with a known relative spectral power distribution  $S(\lambda)$ , usually one of the CIE Standard Illuminants, formed by a black body at a given temperature; illuminant A, for instance, is a black body at 2856 K.

The light of the source passes through a filter which mimics  $V(\lambda)$ . The action of the filter is to subtract for each range of wavelengths  $\Delta\lambda$  centered at  $\lambda_i$  the amount of power determined by  $V(\lambda_i)$ , calculated by (10).

The absolute spectral power responsivity  $s(\lambda)$  (defined as the intensity of the electric current produced in the sensor per watt of incoming radiation, and measured in A/W) of the entire photometer is calibrated using a very precise light source; with these elements, the illuminance responsivity (in A/lx) of the detector is

$$s_v = \frac{A \int_{\lambda_{min}}^{\lambda_{max}} S(\lambda) s(\lambda) d\lambda}{K_m \int_{\lambda_{min}}^{\lambda_{max}} S(\lambda) V(\lambda) d\lambda}.$$
 (20)

In (20) A is the area of a window in front of the detector that limits the amount of light arriving to it; this area is measured with a calibrated instrument. The distance between the light source and the aperture is fixed taking into account that the solid angle subtended from the source to the aperture should have a known value. In this way the responsivity in Ampere per lux could be converted to A/cd.

For a more detailed description of the detector standard of cd, visit the web page at the National Institute of Standards and Technology NIST, Ref. [13]. NIST has also developed a standard for the lumen [14] as well as for other photometric units [15].

### V. CONCLUSIONS

In every experimental or theoretical situation in Optics it is important to understand the type of magnitude (radiometric or photometric) you are interested in, and the aim of the measurement or the calculation. The lack of precision leads to misunderstandings and ambiguities or, in worst cases, errors. Textbooks should be analyzed searching for this kind on errors before using them in a course.

It is important to note that the name of the different magnitudes some times differ from book to book, being necessary to consult the names approved by the standard organizations.

### **BIBLIOGRAPHY**

- [1] Michael Bass (ed.), Handbook of Optics Volume II Devices, Measurements and Properties, 2nd Ed., McGraw-Hill (1995).
- [2] F.J. Low, G.H. Rieke, The Instrumentation and Techniques of Infrared Photometry, Editor: N. Carleton, Methods in Experimental Physics, Academic Press, Volume 12, Part A, 415-462 (1974).
- [3] https://en.wikipedia.org/wiki/Microwave, Accessed 01/07/2019.
- [4] J. Polívka, Int. J. of Infrared and Millimeter Waves, 16, 1593-1672 (1995).
- [5] https://en.wikipedia.org/wiki/Ultraviolet, Accessed 01/07/2019.
- [6] D. R. Grimes, Adv Exp Med Biol. 996,349-360 (2017).
- [7] https://www.nist.gov/programs-projects/photometry, Accessed 07/01/2019.

- [8] https://en.wikipedia.org/wiki/Radiometry, Accessed 01/07/2019.
- [9] E. Gómez-González. Guía básica de conceptos de radiometría y fotometría. http://laplace.us.es/campos/optica/general/opt-guia2.pdf, Accessed 04/30/2019.
- [10] B.N.Taylor, Guide for the use of the International System of Units (SI), NIST Special Publication 811, U.S. Government Printing Office, Washington, DC 20402-9325, 1995.
- [11] G. S. Landsberg, Optics, Mir Ed. Moscow (1983).
- [12] V. L. Díaz-Melian, L. A. Rodríguez, F. Pedroso-Camejo, J. Mieres, Y. de Armas, A. J. Batista-Leyva, E. Altshuler. Accepted for publication, Rev. Cub. Fis. 36, xxx (2019).
- [13] https://www.nist.gov/pml/sensor-science/opticalradiation/ realization-candela, Accessed 02/20/2019.
- [14] https://www.nist.gov/pml/sensor-science/opticalradiation/ realization-lumen, Accessed 02/20/2019.
- [15] https://www.nist.gov/pml/sensor-science/opticalradiation/ realization-related-photometric-units, Accessed 02/20/2019.

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