

ELECTROMAGNETIC ANALOGUE OF A POINT STRUCTURAL PARTICLE

Ll. Uranga¹, M. Y. Ballester A. Martínez and E. Entralgo[†]

Instituto Superior de Tecnología y Ciencia Aplicadas (INSTEC), La Habana, Cuba

ABSTRACT

Starting with the canonical equations of a electromagnetic field and its equivalence with a system of oscillators the fields corresponding to an isolated point structural particle (p.s.p.) and in presence of stationary and homogeneous electric and magnetic fields are obtained. It is showed that the state of the p.s.p. can be obtained through its electromagnetic analogue, neglecting the terms of the subparticles radiation losses.

RESUMEN

En este trabajo, a partir de la equivalencia entre las ecuaciones canónicas del campo electromagnético y un sistema de osciladores, se obtienen los campos electromagnéticos equivalentes a una partícula estructural puntual (p.s.p.) aislada y en presencia de campos eléctricos y magnéticos homogéneos y estacionarios. Se demuestra que el estado de la p.s.p. puede ser representado por su análogo electromagnético si se desprecian los términos que expresan las pérdidas por radiación de las subpartículas.

I. INTRODUCTION

Since the early studies of Schrödinger [1] on building a classical theory of a particle with spin, a lot of work has been carried out in order to develop models and to analyze physical implications of such particle. The main difficulty of the large number of theoretical studies devoted to this matter [2] is that the notion of spin is introduced in mathematical formalisms together with other hidden variables which have no explicit physical meaning. Besides, in the models mentioned in [2], either the particles are not point objects or the theory can be called classical only formally. In the papers [2-5] the theory of the point structural particle (p.s.p.) was developed, that is, a classical theory for a point particle with spin and other variables that have a clear physical meaning.

The straightforward derivation of a relativistic model of a p.s.p. is difficult since it involves relativistic equations of motion of a many particle system. The aim of this work is to present an alternative approach based on the equivalence of a p.s.p. and an electromagnetic field. It allows to use the intrinsic invariance of the electromagnetic field under Lorentz transformations. To this purpose an isolated p.s.p. and an uncharged p.s.p. in presence of homogeneous and stationary electric and magnetic fields are analyzed.

The organization of the paper is the following. In Section II we briefly describe the model of the point structural particle and the general procedure. In Section III the results are presented and discussed. Finally, in Section IV some conclusions are drawn.

II. THE MODEL

A p.s.p. is a cluster of point subparticles of masses m_j , charges e_j , coordinates q_j and velocities dq_j/dt . Due to their interaction they remain confined in small region of space

$$|q_j(t) - q_k(t)| \leq l_0 \quad (1)$$

for arbitrary values of k, j and t .

Such condition leads to describe the p.s.p. through magnitudes that characterize it as a whole: the total mass m , the charge e , center of mass coordinate q and momentum p , besides, total angular momentum J and total energy E . Time evolution of this quantities is determined by an infinite set of coupled differential equations. This equations system is truncated using the small parameters l_0 and

$$Z_j = \frac{e_j}{m_j} - \frac{e}{m} \quad (2)$$

Then, a closed system of equations for the 15 independent variables is obtained [4,6-8]:

$$P = m\dot{q} + \frac{(g - g_0)}{c} S \times E + \frac{2g(g - g_0)}{K} (d\nabla) S \times E,$$

$$\dot{d} = F,$$

E-mail: ¹llinersy@info.isctn.edu.cu

$$\dot{\mathbf{F}} = -\omega_0^2 \mathbf{d} + \kappa \mathbf{E} + (\mathbf{d}\nabla) \left(\mathbf{G} + \frac{\mathbf{e}}{m} \mathbf{E} \right) + \left(\frac{\kappa}{mc} \mathbf{P} + \frac{\mathbf{e}}{mc} \mathbf{F} \right) \times$$

$$\times \mathbf{H} + \frac{\mathbf{e}}{m^2 c} \mathbf{P} \times (\mathbf{d}\nabla) \mathbf{H} + \frac{2g(g-g_0)}{m} \nabla \mathbf{S} \cdot \mathbf{E} \times \mathbf{P},$$

$$\dot{\mathbf{P}} = m\mathbf{G} + e\mathbf{E} + \frac{\mathbf{e}}{mc} \mathbf{P} \times \mathbf{H} + (\mathbf{d}\nabla)^2 \left[\mathbf{G} + \frac{\mathbf{e}}{m} \mathbf{E} \right] +$$

$$+ g\nabla(\mathbf{S}\mathbf{H}) + \frac{2c}{\kappa} g(g-g_0)(\mathbf{d}\nabla)\nabla(\mathbf{S}\mathbf{H}) + \frac{1}{c} \cdot$$

$$\cdot \left\{ \left(\frac{\mathbf{P}}{m} + \frac{\mathbf{e}}{m\kappa} \mathbf{F} \right) \times (\mathbf{d}\nabla) \mathbf{H} + \frac{\mathbf{e}}{2m^2 \kappa} \mathbf{P} \times (\mathbf{d}\nabla)^2 \mathbf{H} \right\} +$$

$$+ \frac{2g(g-g_0)}{m\kappa} \nabla(\mathbf{d}\nabla) \mathbf{S} \cdot \mathbf{E} \times \mathbf{P},$$

$$\dot{\mathbf{S}} = \mathbf{S} \times \left\{ g\mathbf{H} + \frac{2c}{\kappa} g(g-g_0)(\mathbf{d}\nabla) \mathbf{H} \right\} + (g-g_0) \cdot$$

(3)

$$\cdot \left\{ \frac{1}{mc} \mathbf{S} \times \mathbf{E} \times \mathbf{P} + \frac{2g}{m\kappa} (\mathbf{d}\nabla) \times \mathbf{E} \times \mathbf{P} \right\}$$

with energy

$$E = \frac{F^2}{2\kappa} + \frac{\omega_0^2}{2\kappa} d^2 + W_0 + \frac{P^2}{2m} + m\phi + e\varphi - dE +$$

$$+ \frac{1}{2\kappa} (\mathbf{d}\nabla)^2 \left(\phi + \frac{\mathbf{e}}{m} \varphi \right) - g\mathbf{S}\mathbf{H} - \frac{(g-g_0)}{mc} \mathbf{S} \cdot \mathbf{P} \times \mathbf{E} -$$

(4)

$$- \frac{2g(g-g_0)}{m\kappa} (\mathbf{d}\nabla) (\mathbf{S} \cdot \mathbf{P} \times \mathbf{E})$$

where $\phi(r), \varphi(r), A(r)$ are the potentials of the external gravitational, electric and magnetic fields and \mathbf{G}, \mathbf{E} and \mathbf{H} are their respective intensities.

Although the relativistic invariance of electromagnetic fields is restricted to those ones which obey Lorentz gauge, as a first approach we employ Coulomb gauge. Using a convenient normalization canonical equations of such field become equivalent to a system of harmonic oscillators [9]. We use the inverse procedure to assign to each system its corresponding field.

III. RESULTS AND DISCUSSION

A. Isolated p.s.p.

The energy of an isolated p.s.p. is:

$$E = \frac{F^2}{2\kappa} + \frac{\omega_0^2}{2\kappa} d^2 + \frac{P^2}{2m} \quad (5)$$

being ω_0 the proper frequency.

We propose an electromagnetic field that satisfies both Maxwell equations and Coulomb gauge.

$$\mathbf{A} = C \left[-f \cos(\kappa r) + \omega_0 d \sin(\kappa r) \right] \quad (6)$$

If we choose

$$C = \sqrt{\frac{4\pi}{L^3}} \frac{1}{k_0 \sqrt{\kappa}} \quad (7)$$

the energy of the field will be equal to the internal energy of the p.s.p.

B. Uncharged p.s.p. in presence of homogeneous and stationary electric and magnetic fields

For an uncharged p.s.p. in an homogeneous electric field we obtain:

$$\dot{\mathbf{f}} = -\omega_0^2 \mathbf{d} + \kappa \mathbf{E} \quad (8)$$

and with the electromagnetic field

$$\mathbf{A} = \sqrt{\frac{4\pi}{L^3}} \frac{1}{k\sqrt{\kappa}} \left[-f \cos(\kappa r) + \omega_0 \left(d - \frac{\kappa \mathbf{E}}{\omega_0^2} \right) \sin(\kappa r) \right] \quad (9)$$

the energy inside the normalization box is

$$E = \frac{\omega_0^2 d^2}{2\kappa} + \frac{f^2}{2\kappa} - E d + \frac{\kappa E^2}{2\omega_0^2} \quad (10)$$

It is the same that the internal energy of the particle [10] except for the last term. This term is proportional to the charge of the subparticles and the electric field intensity. It is related to radiation losses of the subparticles [9], which is implicitly included in the Maxwell equations and its not considered in the model of the p.s.p.

In the case of an uncharged p.s.p. in an homogeneous magnetic field the relevant equations of motion of the internal degrees of freedom are:

$$\begin{aligned} \ddot{f}_{\parallel} &= \omega_0^2 f_{\parallel} \\ \ddot{f}_{\perp} &= -\omega_1^2 f_{\perp} \\ \ddot{x} &= -\omega_2^2 x \end{aligned} \quad (11)$$

where

$$x = -\omega_0 \kappa \frac{\sqrt{2gSH}}{gH} S_{\perp} \quad (12)$$

f_{\parallel} and f_{\perp} are respectively the components of the dipolar velocity parallel and perpendicular to the spin axis.

$$\begin{aligned} \omega_1^2 &= \omega_0^2 - \frac{\kappa H^2}{m_c^2}, \\ \omega_2^2 &= g^2 H^2 \end{aligned} \quad (13)$$

Following the same procedure we choose

$$\begin{aligned} A = \sqrt{\frac{4\pi}{L^3}} \frac{1}{k_0 \sqrt{\kappa}} = & \left\{ -f_0 \cos(k_0 r) + \frac{1}{\omega_0} \dot{f}_0 \sin(k_0 r) - \right. \\ & - f_1 \cos(k_1 r) + \frac{1}{\omega_1} \dot{f}_1 \sin(k_1 r) - x \cos(k_2 r) + \\ & \left. \frac{1}{\omega_2} \dot{x} \sin(k_2 r) \right\} \end{aligned} \quad (14)$$

yielding an energy

$$E = \frac{w_0^2 d^2}{2\kappa} + \frac{f^2}{2\kappa} - gSH - \frac{(f \times H)^2}{2mc^2 \omega_0^2} + \frac{(p \times H)^2}{2mc^2 \omega_0^2} + \frac{p(d \times H)}{mc} \quad (15)$$

The first three terms represent the internal energy of the p.s.p. [10] while the later ones have the same radiation origin that in the case of homogeneous electric field.

IV. CONCLUSIONS

In this paper we proposed classical electromagnetic fields which are equivalent to an isolated p.s.p. and a p.s.p. in presence of homogeneous and stationary electric and magnetic external fields. This allows to obtain the equations of motion of a p.s.p. from that ones of the analogue system and represents a first step towards the building of a relativistic model of a classical particle with spin.

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